Macroscopic Models of Thin Conductive Layers: Systematic Evaluation for Microwave Heating and Shielding Applications

Malgorzata Celuch#1, Konrad Wilczynski$2, Marzena Olszewska-Placha#3
#QWED Sp. z o.o., Poland
$Faculty of Physics, Warsaw University of Technology, Poland
1mceluch@qwed.eu, 2Konrad.Wilczynski@pw.edu.pl, 3molszewska@qwed.eu

Abstract—**This paper aims at discussing an accuracy of macroscopic models of thin conductive layers used in electromagnetic modelling for the purpose of microwave heating and shielding applications. Thin conductive layers, characterised by their surface resistance, are typically replaced by a surrogate layer of higher thickness to decrease computer efforts of 3D discrete modelling methods and make the problem resolvable in a reasonable time. Experiments conducted in this work with the aid of analytical calculations and conformal finite difference time domain (FDTD) method, allow addressing sensitivity of the considered model to surrogate layer thickness, angle of incidence, changes of characteristic impedance, etc. Practical modelling guidelines for both heating and shielding applications are formulated.**

Keywords—**Susceptors, active packaging, conductive coatings, microwave heating, EM shielding, electromagnetic modelling, microwave ovens, FDTD.**

I. MOTIVATION AND BACKGROUND

Thin conductive layers are encountered in extremely diverse areas of microwave (MW) technology. In our daily life they are known as Susceptors in microwaveable food packages [1],[2],[3], which serve to enhance local dissipation of MW power, resulting in desired oven functions such as corn popping, or producing specific sensory effects such as a crispy pizza crust. At the other extreme of applications, the coating of carbon-fiber layers with thin layers of e.g. silver nano-particles has been reported to enhance electrical conductivity of Carbon Fibre Reinforced Polymers (CFRPs), without increasing the weight [4]. This seems a vital development for aircraft industry and in-flight safety, since contemporary aircrafts utilise CFRPs for more than 50% of their structural weight to reduce fuel consumption. At the same time, an adequate damage protection is necessary considering that an average aircraft in service is exposed to 1-2 lightning strikes a year [4].

Nowadays it is impossible to envisage MW engineering without the support of electromagnetic (EM) modelling, and 3D discrete methods such as FDTD [6] or FEM [7] come closest to allow virtual prototyping. However, thin metal layers are a challenge for those methods. Surface impedance boundary condition (SIBC) models available for lossy metals in FDTD [6] are not applicable, since they miss the semi-transparency effect for thicknesses less than the penetration depths. With thicknesses by orders of magnitude less than the operating wavelength (see Table 1) volumetric meshing also becomes impractical. This creates a need for developing dedicated macroscopic models.

A related issue is a difficulty in characterising the material parameters of conducting films. Since their layers are thin, surfaces tend to be rough, and the use of patterns in the lateral plane is often necessary (e.g. to ensure breaking of popcorn Susceptor at a certain temperature), the only practically measurable parameter is sheet resistance (in Ohm per square) [8]. For this purpose, the use of split-post and single-post dielectric resonators has been reported in e.g. [8],[9].

Based on those observations, it has been proposed that a MW Susceptor can be adequately modelled by a scaled (thicker) dielectric layer of proportionally scaled (decreased) conductivity, such that the sheet resistance is maintained [8],[10]:

$$R_s = (d_0 \sigma_0)^{-1} = (d\, \sigma)^{-1} \tag{1}$$

where σ_0, d_0 denote parameters of the original Susceptor (not necessarily separately known), while σ, d are the scaled values. Such modelling has been adapted in other applications including MW absorbers [9] and is recommended in general-purpose FDTD software applications, e.g [12].

In the context of MW oven and MW food packages design, the main questions concerning not only the accuracy, but also the physical sense of the surrogate models after eq. (1) are:

a) How much can one scale the original Susceptor, without corrupting its power absorption properties? Will a food package developed based on the surrogate model perform equally well in reality?

b) Even if the scaling by eq. (1) intuitively holds for normal incidence of TEM waves, does it also hold for arbitrary angles?

c) The scaling (1) changes characteristic impedance and wavelength in the modelled Susceptor. Are those changes neutral to the model behaviour?

d) It is known that metal surfaces provide polarisation-independent reflection coefficient, while reflections from lossy dielectric are polarisation-dependent. Do we know that the scaling does not therefore change field distribution in MW oven, above and below the Susceptor?

e) Can we keep the scaled material isotropic, as in eq. (1), or should we only scale the in-plane elements of conductivity tensor, as proposed in [8]?

f) When the results of EM modelling diverge from measurements, to what extent is it be due to the analytical scaling concept of (1), and to what extent - due to its numerical implementation in FDTD or FEM?

Table 1. Examples of thin metal films for MW heating (1-6) and shielding (7-10) applications.

#	Description	Source of data	Rs (Ω/sq.)	Physical thickness
1	Pizza before use	@ [8]	104.2	<<1 μm
2	Pizza after use	@ [8]	21500	<<1 μm
3	Popcorn before use	@ [8]	34.96	<<1 μm
4	Popcorn after use	@ [8]	26500	<<1 μm
5	PET/aluminum at 160..200 deg	[1], Fig. 9.8	ca. 100	-
6	PET/aluminum at 240 deg	[1], Fig. 9.8	ca. 400	-
7	Ti film	[9] Sec. II	200	10 nm
8	Graphene nano-platelets ink	[11]	24	23±0.5 μm
9	Al film	[9] Sec. II	0.1	300 nm
10	Conductive coatings & paints	[5]	0.001..11	0.4 – 1.5 mil

While some of the above questions have been previously selectively addressed, in this paper - for the first time, to our knowledge - we approach them in a systematic manner. We conduct extensive analytical calculations as well as computer simulations with conformal FDTD method [12], also indicating the related limitations of the classical stair-case FDTD.

II. POWER ABSORPTION BY MACROSCOPIC MODELS

Table 1 shows selected examples of thin conductive films for MW heating (1-6) and absorbing (7-10) applications. As presented, Susceptors in MW food packages typically have sheet resistance of 30-400 Ω. Such values have been proven to provide the highest absorption rates at normal incidence [1]. However, the normal incidence is not representative for practical MW ovens. As discussed in [13], fields in a statistically typical multimode oven can be represented by an equivalent incidence angle of 47 deg and for well-performing ovens this angle increases to 60..80 deg. In Fig. 1 we exemplify that the fastest heating of beef above the room temperature occurs with TM incidence at 82 deg.

In Fig. 2 we provide relative power absorption maps for typical values of sheet resistance corresponding to (after Table 1): 1 Ω - absorbing panels, 35 & 105 Ω - popcorn and pizza Susceptors at room temperature, 400 Ω - food Susceptors breaking at above 240 deg, 20000Ω - Susceptors after use. They confirm that the values of Rs=30-400 Ω provide best absorption for arbitrary field patterns, although higher-resistance sheets perform best at normal incidence and lower-resistance - at high incidence angles. Finally, the scaling after eq. (1) seems neutral to power absorption, for the surrogate layer thickness of up to 1 mm, which is more clearly seen in the cuts through the maps, shown in Fig. 3.

The maximum surrogate thickness of 1mm can therefore be recommended for practical Susceptor modelling in MW ovens. At 2.45 GHz it corresponds to ca. 0.01 wavelength in air and 0.1 λ in high permittivity foods such as beef (Fig. 1).

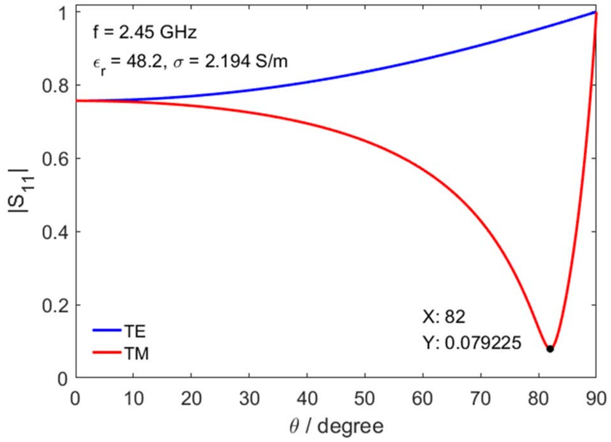

Fig. 1. Reflection coefficient for TE and TM -polarised waves impinging from air onto beef (at room temperature).

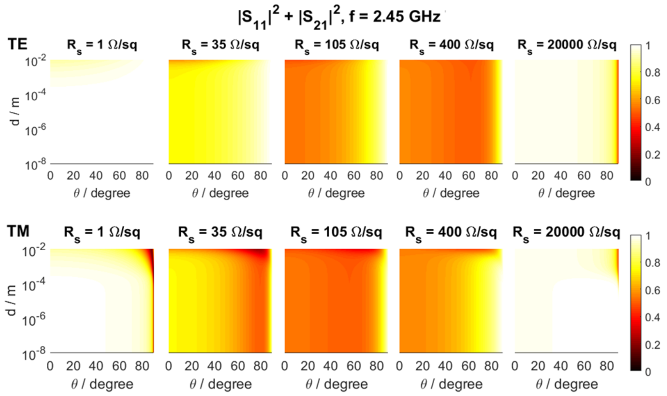

Fig. 2. Maps of relative absorbed power for varying angle of incidence *θ* and thickness *d* of the surrogate lossy layer.

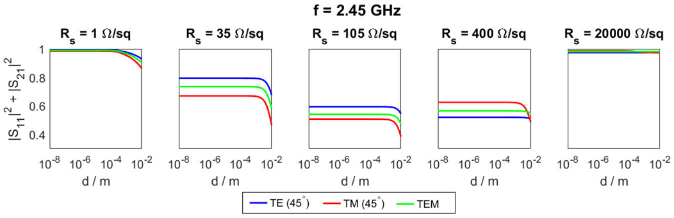

Fig. 3. Vertical cuts through the maps of Fig. 2, for normal incidence (TEM) and TE, TM polarisation at 45 deg.

III. REFLECTION AND TRANSMISSION

In Fig.4-7 we now focus on the incidence angles of 0, 45 and 80 deg (selected for the reasons above) and separately plot the amplitudes and phases of the reflection and transmission coefficients. The presented results confirm that a surrogate layer model with a layer thickness up to 1 mm, does not deteriorate EM modelling results, either in terms of reflection, transmission or power dissipated in the Susceptor. This accuracy criterion remains valid for incidence angles other than 0 deg. Phases of the S-parameters are slightly more sensitive, but a few degrees phase shift between TE and TM waves will

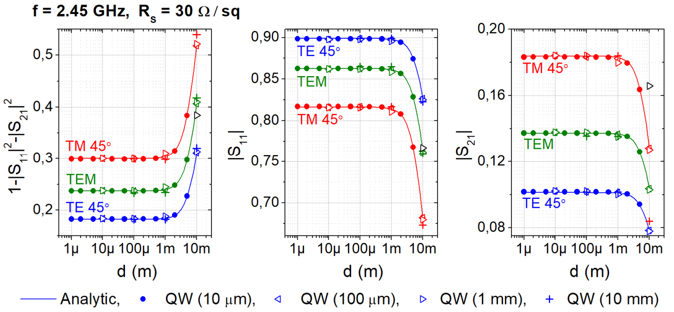

Fig. 4. Relative absorbed power, reflection, and transmission for waves incident perpendicularly and at 45 deg. onto a 1μm-thick Susceptor, modelled with (1) by a surrogate layer of thickness increasing to 10 mm: analytical scaling results compared to conformal FDTD simulations with different basic cell size. Black ► is a check point for anisotropic model (2)(3) of TM case.

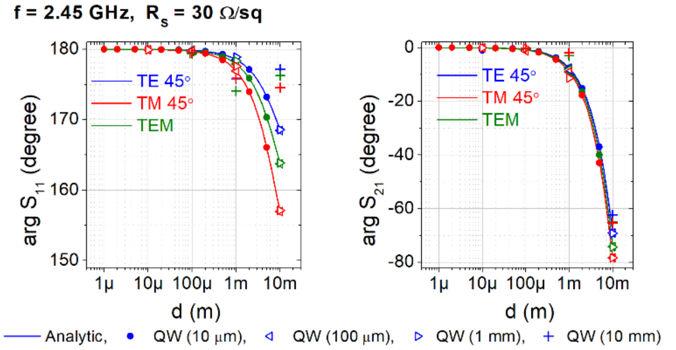

Fig. 5. Phases of reflection and transmission coefficients, for waves incident perpendicularly and at 45 deg., onto a 1μm-thick Susceptor, modelled with (1) by a surrogate layer of thickness *d* increasing to 10 mm: analytical scaling results compared to conformal FDTD simulations with different basic cell size.

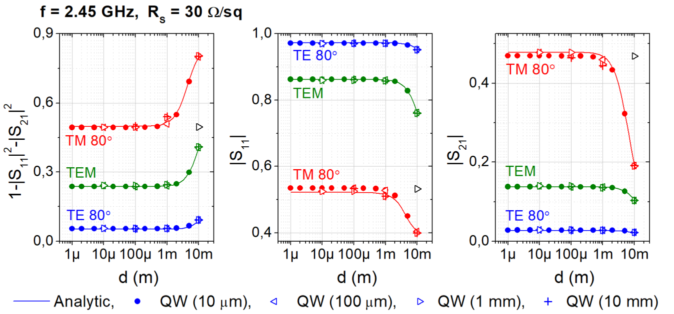

Fig. 6. Relative absorbed power, reflection, and transmission for waves incident perpendicularly and at 80 deg. onto a 1μm-thick Susceptor, modelled with (1) by a surrogate layer of thickness increasing to 10 mm: analytical scaling results compared to conformal FDTD simulations with different basic cell size. Black ► is a check point for anisotropic model (2)(3) of TM case.

not visibly change the polarisation of total field below and above the Susceptor.

Analytical considerations in Fig.4-7 are confirmed by conformal FDTD simulations with different meshing. The normal incidence is modelled by TEM wave propagation in a paralle-plate line, while the oblique incidence of TE and TM wave is modelled as TE11 and TM11 modes in a square waveguide of appropriately set dimensions. The FDTD results (points) coincide with the analytical ones (continuous lines) for meshing as coarse as 10 mm for magnitudes, and 1 mm for phases. Note that the applied conformal FDTD allows rigorous subcellular approximation of 0.01 cell thick surrogate, while in the stair-case FDTD the surrogate must occupy a full cell.

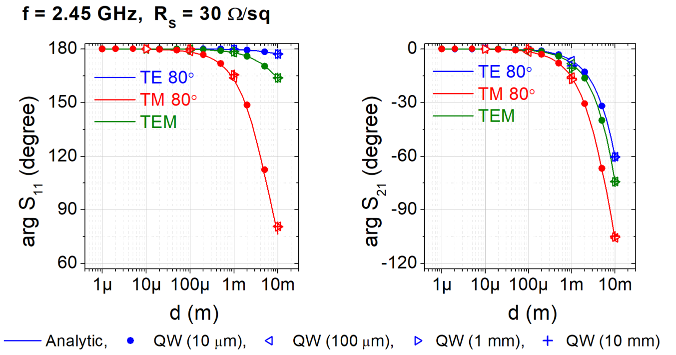

Fig. 7. Phases of reflection and transmission coefficients, for waves incident perpendicularly and at 45 deg., onto a 1μm-thick Susceptor, modelled with (1) by a surrogate layer of thickness *d* increasing to 10 mm: analytical scaling results compared to conformal FDTD simulations with different basic cell size.

IV. ANISOTROPIC MODELS

Equation (1) assumes that surrogate material is of isotropic type. Different approach is presented in [8], wherein anisotropy of the material model and thereby scaling of only in-plane elements of conductivity tensor is proposed. Therefore, the scaling is described by:

$$(d_0\,\sigma_{T0})^{-1} = (d\,\sigma_T)^{-1} \tag{2}$$

$$\sigma_n = 0 \tag{3}$$

where $\sigma_{\tau 0}$ denotes conductivity tensor of the original Susceptor, while σ_τ is a tensor of scaled conductivity values, with σ_n element normal to the Susceptor's plane. In Table 2 we compare FDTD anisotropic model to analytical and FDTD isotropic models, for 1 μm thick Susceptor scaled to 1 mm and 10 mm, irradiated with TM polarised wave (anisotropy is irrelevant to TE and TEM). The anisotropic model remains closer to the original Susceptor, especially for big d = 10 mm (bold values in Table 2, marked ► in Fig.4,6), which indicates that the surrogate thickness can potentially be extended beyond the previously indicated 1 mm.

Note however, that the advantage of the anisotropic model holds only for TM waves and thick surrogates; in problems that involve curved conductive sheets, where anisotropy definition

Table 2. Reflection coefficient for TM wave impinging onto a 1μm -thick Susceptor and its surrogate models scaled to 1mm and 10mm: analytical values are given for isotropic model while conformal FDTD ([12]) modelling results with basic cell size of 1 mm are given for both isotropic and anisotropic model.

|S11|, TM, f= 2.45 GHz						
d (μm)	**45°**			**80°**		
	Anal. Iso.	**QW Iso.**	**QW Aniso.**	**Anal. Iso.**	**QW Iso.**	**QW Aniso.**
1	**0.81621**	-	-	**0.52166**	-	-
1000	0.81388	0.81065	0.81525	0.51563	0.51091	0.53402
10000	0.68099	0.67966	***0.76690***	0.40892	0.40034	***0.53292***

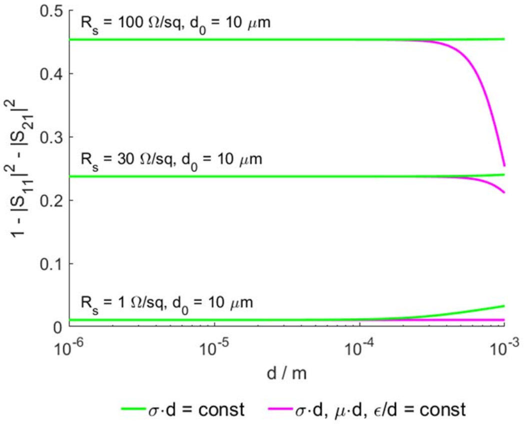

Fig. 8. Relative absorbed power for scaled models of 10 µm -thick Susceptors, of three values of sheet resistance: 1 Ω/sq., 30 Ω/sq., and 100 Ω/sq.

in FDTD would not be straightforward, the isotropic model can be applied with good accuracy, as discussed in Section IV.

V. IMPEDANCE - CONSERVING MODELS

In this Section we propose a new scaling technique, which becomes an alternative to that proposed by Eq. (1). It is based on impedance conserving models. Therefore, when eq. (1) is applied, we also modify material permeability to conserve intrinsic impedance of the Susceptor, which under the assumption of:

$$\tan\delta = \sigma\,(\omega\varepsilon)^{-1} \tag{4}$$

reads:

$$Z = \sqrt{\frac{j\omega\mu}{\sigma}} \tag{5}$$

and we thus we apply:

$$d_0\,\mu_{r0} = d\,\mu_r \tag{6}$$

where μ_{r0} denotes relative permeability parameter of the original Susceptor, while μ_r is the one of the surrogate. Then to satisfy FDTD stability criterion in the surrogate we also scale permittivity ε_{r0} to ε_r following:

$$d_0 / \varepsilon_{r0} = d / \varepsilon_r \tag{7}$$

where ε_{r0} denotes relative permittivity parameter of the original Susceptor, while ε_r is the one of the surrogate material.

In Fig. 8 we provide a comparison of power absorbed by a 10 µm thick Susceptor of different surface resistance, modelled with surrogate layers based on the two considered scaling models. The new impedance-conserving model involving Eq. (6)(7) allows accurate scaling to thicker surrogates in the case of low sheet resistance relevant to MW shields; in the case of lower-conductivity it diverges from the original at lower surrogate thicknesses, as relation (4) is faster violated. The original conductivity-only scaling is therefore recommended for Susceptors in MW food packages.

VI. CONCLUSIONS

In this paper a systematic evaluation of macroscopic models of thin conductive layers in a form of surrogate layer with significantly higher thickness is presented. The calculations are conducted at different incidence angles and for different EM wave polarisations, analytically and via FDTD simulations. In specific cases, we demonstrate advantages of slightly modified scalings. For example, anisotropic surrogates are shown advantageous for TM polarisation and thick surrogates. The presented discussion specifically refers to the modelling of Susceptors for MW heating applications. However, our reasoning is applicable to shielding and absorbing panels, for which a new impedance-conserving is recommended.

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